Understanding the effect of the economic crisis on the public education performance through a Hicks-Moorsteen index

Juan Aparicio

Center of Operations Research, University Miguel Hernandez of Elche, Av.de la Universidad s/n, 03202 Elche, Spain

Laura López-Torres

Department of Business Organization, University of Granada, Campus Cartuja s/n, 18071, Granada, Spain

Daniel Santín

Department of Applied Economics VI, Complutense University of Madrid, Campus de Somosaguas, 28223 Madrid, Spain

Abstract

The present study offers an alternative total factor productivity (TFP) change index, the Hicks–Moorsteen index, defined as a ratio of a Malmquist output over a Malmquist input index, to assess productivity changes. In contrast to the well-known Malmquist index, the standard Hicks-Moorsteen index always leads to feasible results; it does have a full TFP interpretation and satisfies the determinateness axiom. We illustrate the benefits of this index in a sample of vulnerable Spanish public primary schools covering the period between 2009 and 2014, when budgetary constraints started in the public sector due to the financial crisis. The results reveal that in times of crisis schools improved their total factor productivity as a result of raising academic achievement and losing resources.

Keywords

Productivity index, Hicks-Moorsteen, Public Schools, Economic crisis, education

1. Introduction

In the wake of the global economic crisis, countries face the challenges of making public finances sustainable. Likewise, the way public funding is spent is more closely scrutinized in times of austerity (Bel et al., 2010). Publicly funded sectors are under pressure to deliver more for less and the education sector is not an exception. At this juncture, any action aimed at assessing performance in the public sector is a priority of economic policy. Performance measurement is important not only in the private sector, but also in the public sector since it can highlight strengths and weaknesses in current practices, reveal directions for improvement, and ultimately may lead to better use of the resources spent on providing public services (Asmild et al., 2012).

Since the pioneering work by Bessent and Bessent (1980), Charnes *et al.* (1978, 1981) and Bessent *et al.* (1982), empirical studies on efficiency in education have grown in importance. Since its inception, this body of literature has brought findings that contribute to the knowledge and better understanding of the educational factors that influence students' development. It also provides useful information for public administration decision makers.

Although there are many empirical approaches and methods for measuring efficiency in education (see De Witte and López-Torres, 2015 and Johnes, 2015 for a review), frontier methods is the gold standard approach used in two forms: non-parametric [Data Envelopment Analysis (DEA), Free Disposal Hull (FDH), order-m frontiers] and parametric [Stochastic Frontier Analysis (SFA)] methods.[1] Frontier methods faithfully illustrate the essential characteristics of measuring efficiency and are worth for benchmarking decision making units (DMUs). A myriad of papers have applied these methods in different educational production units, however, it is worth noting that a relatively lower body of literature has applied a dynamic index in order to expand the findings obtained in the static scenario to reveal productivity changes over time. A possible explanation for the lack of empirical productivity studies is the difficulty of gathering panel databases at school level with standardized outcomes from the educational administration.

[1] A review of the advantages and shortcomings of different frontier analysis techniques can be found in Fried *et al.* (2008).

Assessing the drivers of productivity growth is important for education policy. Their identification allows monitoring schools and pointing out best and worst practices, a valuable information that can guide policymakers in their decisions. Following this target, the literature has sought to decompose various measures of productivity growth into components of technical change and efficiency change[2] through ratio-based productivity indexes. One of the most used approaches, and probably the standard in education, is the Malmquist index introduced as a theoretical index by Caves et al., 1982. Some empirical examples are the works by Maragos and Despotis (2004), Agasisti and Pérez-Esparrells (2010), Thanassoulis et al. (2011), Essid et al. (2014) and Johnson and Ruggiero (2014).

Nevertheless, in spite of its advantages, and following Bjurek et al. (1998), the recent literature has clearly established that the Malmquist productivity index has three important drawbacks. First, the Malmquist index does not have a direct total factor productivity (TFP) interpretation. Second, the Malmquist index could lead to infeasibility problems. Third, the Malmquist productivity index must be always calculated assuming constant returns to scale, regardless the returns to scale exhibited by the DMUs in the periods of time under assessment. This fact prevents the use of the variable returns to scale assumption even when the context suggests that it is the hypothesis that must be assumed in the analysis.

In this scenario, Bjurek (1996) offers an alternative Hicks–Moorsteen TFP (HMTFP) index, defined as a ratio of a Malmquist output-oriented index over a Malmquist input-oriented index (O'Donnell, 2010, 2012a). The Hicks–Moorsteen index satisfies the determinateness axiom, as conjectured by Bjurek (1996) and demonstrated in Briec and Kerstens (2011) under mild conditions (i.e., mainly strong disposability of inputs and outputs)[3]. The HMTFP index is therefore presented as an ideal tool to obtain valid conclusions for the correct monitoring of DMUs in terms of resource allocation, as it is a ratio of an output quantity index to an input quantity index (O'Donnell, 2012a). In addition, because of the manner of its construction, it can be easily decomposed into meaningful measures of technical change and efficiency change.

[2] See Färe et al. (1998) and Grosskopf (2003) for historical overviews.
[3] We will review the advantages and shortcomings of both indexes in Section 2.

While the Malmquist productivity index is very popular, the HMTFP index has so far found rather limited use in applied research. Up to our knowledge a complete list of empirical applications includes the paper by Bjurek et al. (1998) that represents the first empirical application of the Hicks–Moorsteen index on the electricity supply; Zaim (2004) who applied the index in state manufacturing companies; Nemoto and Goto (2005) that develop the index in a parametric context in Japanese prefectures; Epure et al. (2011), Arjomandi et al. (2012), Arora and Arora (2012, 2013) and Yi (2016) with applications in the banking industry; Hoang (2011), O'Donnell (2012a, 2012b) and Ang and Kerstens (2017) that applied the model in the agricultural sector; Simões and Marques (2012) use the HMTFP index to test the influence of regulation on the productivity of Portuguese waste utilities. Further empirical application of this index can be found in Kerstens and Van de Woestyne (2014) that use two datasets, one from French fruit producers and another one from Chilean hydro-electric power plants, and Medal-Bartual et al. (2015) who assess productivity changes in Spanish Ports. As can be seen, none one of the abovementioned papers have applied the Hicks–Moorsteen productivity index in the educational context.

In terms of empirical contributions, this paper represents the first attempt of using the HMTFP indicator in the educational context for highlighting its relative advantages for developing a monitoring system to be used by policy makers.

Particularly, the current paper contributes to the existing literature by applying a HMTFP index in a sample of vulnerable schools from a Spanish region, Catalonia, whose public education sector was drastically affected by the economic crisis and budgetary constraints. In this environment, it is crucial to be able to study how the economic crisis affected TFP changes in recent years in the educational sector. In this sense, the HMTFP index allow us to analyze output and input changes together with a technological and efficiency decomposition to identify best practices and resource mismanagement. In this way, we apply the HMTFP index in a sample of 298 public schools from Catalonia using a rich balanced data set covering five academic years, from 2009-2010 to 2013-2014,[4] representing a suitable period to analyze the antecedents and consequences of the economic crisis on productivity changes. We put

[4] For simplicity, we will refer to each academic year by mentioning the first natural year (for instance, 2009 to refer to 2009/2010).

special emphasis in those schools located in poorest neighborhoods, as they are the most affected by budget cuts.

The outline of the paper is as follows. Section 2 details the methodological approach used in this paper. In Section 3 we describe the data and variables we use for the study. Section 4 summarizes the empirical results and finally, Section 5 concludes.

2. Methodology

In this section we briefly revise the definition and main features of the Hicks–Moorsteen Total Factor Productivity (HMTFP) index, which is the approach that we use propose to determine productivity changes over time in education. Nevertheless, we first need to introduce some notation.

Let as assume that we focus our attention on two periods of time, denoted as t and $t+1$. For period t, let us define an input vector as $x^t \in R_+^m$ and an output vector as $y^t \in R_+^s$. We assume that for each period we have observed n DMUs with different inputs and outputs, denoted, for period t, as $\left(x_j^t, y_j^t\right)$, which come from a reference technology $T^t = \left\{\left(x^t, y^t\right) \in R_+^m \times R_+^s : x^t \text{ produces } y^t\right\}$. Particularly, T^t is estimated in DEA as $\left\{\left(x^t, y^t\right) \in R_+^m \times R_+^s : \sum_{j=1}^{n} \lambda_j x_j^t \leq x^t, \sum_{j=1}^{n} \lambda_j y_j^t \geq y^t, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0\right\}$ under the Variable Returns to Scale (VRS) assumption (Banker et al., 1984) and as $\left\{\left(x^t, y^t\right) \in R_+^m \times R_+^s : \sum_{j=1}^{n} \lambda_j x_j^t \leq x^t, \sum_{j=1}^{n} \lambda_j y_j^t \geq y^t,\ \lambda_j \geq 0\right\}$ under Constant Returns to Scale (CRS) (Charnes et al., 1978).

The measurement of productivity change over time using frontier methods has claimed considerable attention in the literature that centers on the assessment of economic performance of DMUs. The most popular approach to evaluate productivity change, when market prices are not available, is the Malmquist productivity index introduced by Caves et al. (1982) and popularized by Färe et al. (1994) that made it empirically tractable in a DEA framework, also allowing for the decomposition of productivity change into efficiency and technical changes. The Malmquist index is a ratio-based index that uses Shephard (1953) distance functions to represent technology and, in its most popular forms, adopt either an input contraction or an output expansion perspective.

In particular, under the input reduction perspective, the ‘adjacent’ Malmquist input based productivity index is defined as the geometric mean of two Malmquist input based productivity indices as introduced by Caves et al. (1982):

$$M_I^{t,t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \left[\frac{D_I^t\left(x^t, y^t\right)}{D_I^t\left(x^{t+1}, y^{t+1}\right)} \cdot \frac{D_I^{t+1}\left(x^t, y^t\right)}{D_I^{t+1}\left(x^{t+1}, y^{t+1}\right)}\right]^{1/2}, \qquad (1)$$

where $D_I^k\left(x^h, y^h\right) = \sup\left\{\tau : \left(\frac{x^h}{\tau}, y^h\right) \in T^k\right\}$ is the Shephard input distance function calculated from the point $\left(x^h, y^h\right)$, $h = t, t+1$, to the frontier of the technology at time k, $k = t, t+1$.

The decomposition of (1) in an efficiency change component and a technical change component is given as

$$M_I^{t,t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \underbrace{\frac{D_I^t\left(x^t, y^t\right)}{D_I^{t+1}\left(x^{t+1}, y^{t+1}\right)}}_{\text{Efficiency Change}} \underbrace{\left[\frac{D_I^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_I^t\left(x^{t+1}, y^{t+1}\right)} \cdot \frac{D_I^{t+1}\left(x^t, y^t\right)}{D_I^t\left(x^t, y^t\right)}\right]^{1/2}}_{\text{Technical Change}}. \qquad (2)$$

A value of $M_I^{t,t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) > 1$ indicates an improvement in productivity from period t to period $t+1$, $M_I^{t,t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) < 1$ a decline and $M_I^{t,t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = 1$ an unchanged productivity growth. The same can be applied to each of its components.

Likewise, it is possible to define and decompose an ‘adjacent’ Malmquist output based productivity index grounded exclusively on the Shephard output distance function:

$$D_O^k\left(x^h, y^h\right) = \inf\left\{\rho > 0 : \left(x^h, \frac{y^h}{\rho}\right) \in T^k\right\}. \qquad (3)$$

Although the Malmquist index may be interpreted as a measure of total factor productivity change (TFPC) over time, it must not be seen as a Total Factor

Productivity (TFP) measure. In a multidimensional context, TFP is usually defined as the ratio of an aggregate output to an aggregate input. This definition naturally leads to TFP indexes that can be expressed in terms of the ratio of an output quantity index over an input quantity index. Accordingly, TFPC can be defined as the ratio of the corresponding TFP at period $t+1$ to the corresponding TFP at period t. Unfortunately, the Malmquist index cannot be expressed as a TFP index following the above definition (O'Donnell, 2012a).

Moreover, the Malmquist index and its technical change component are also criticized for not fulfilling the determinateness axiom. This drawback is related to the fact that the Shephard distance function can produce infeasibility results when mix periods are evaluated, i.e. when $k \neq h$ in $D_I^k\left(x^h, y^h\right)$ and $D_O^k\left(x^h, y^h\right)$. In particular, this problem is especially associated with the assumption of VRS for the technology of reference.

Additionally, Grifell-Tatjé and Lovell (1995) showed that the Malmquist index in (1) must be always calculated assuming CRS, regardless the actual form of the technology underlying the DMUs' activity in periods t and $t+1$, if we want the Malmquist index to coincide with the traditional measure of productivity change when we work in two dimensions: $\left(y^{t+1}/x^{t+1}\right)/\left(y^t/x^t\right)$. Other authors came to the same conclusion for the general context (multiple inputs and outputs) (see, e.g., Ray and Desli, 1997, Balk, 2001 and Lovell, 2003), suggesting the use of the CRS assumption for the calculation of the Malmquist index. This fact prevents the use of the VRS assumption even when the context suggest that this may be the returns to scale exhibited by the data (e.g., when DMUs present very different sizes in labor and other inputs or when we are aware of the presence of the law of diminishing returns in the analyzed sector).

With the aim of overcoming all these weaknesses of the traditional Malmquist index, the Hicks–Moorsteen Total Factor Productivity (HMTFP) index was introduced[5]. The HMTFP index is defined as a ratio of an aggregate output quantity over an aggregate input quantity index. More precisely, it measures the change in output quantities in the output direction and the change in input quantities in the input direction, instead of exclusively adopting an input or output perspective as the Malmquist index does. The definition of the HMTFP index follows.

[5] The index was discussed by Bjurek (1996) but was attributed by Diewert (1992) to Hicks (1961) and Moorsteen (1961).

$$HM^{t,t+1}\left(x^t,y^t,x^{t+1},y^{t+1}\right)=\frac{QI^{t,t+1}\left(x^t,y^t,x^{t+1},y^{t+1}\right)}{XI^{t,t+1}\left(x^t,y^t,x^{t+1},y^{t+1}\right)}=$$

$$\frac{\left[\dfrac{D_O^t\left(x^t,y^{t+1}\right)}{D_O^t\left(x^t,y^t\right)}\cdot\dfrac{D_O^{t+1}\left(x^{t+1},y^{t+1}\right)}{D_O^{t+1}\left(x^{t+1},y^t\right)}\right]^{1/2}}{\left[\dfrac{D_I^t\left(x^{t+1},y^t\right)}{D_I^t\left(x^t,y^t\right)}\cdot\dfrac{D_I^{t+1}\left(x^{t+1},y^{t+1}\right)}{D_I^{t+1}\left(x^t,y^{t+1}\right)}\right]^{1/2}}. \qquad (4)$$

The characteristics of the HMTFP index solve, in general, the limitations of the traditional Malmquist index. First, it can be trivially interpreted like a TFPC as a ratio of an aggregated output change index $QI^{t,t+1}\left(x^t,y^t,x^{t+1},y^{t+1}\right)$ to an aggregated input change index $XI^{t,t+1}\left(x^t,y^t,x^{t+1},y^{t+1}\right)$. Second, the HMTFP index is always feasible, i.e., it satisfies the determinateness axiom under mild conditions (see Briec and Kerstens, 2011). It is due to the fact that all the input distance functions included in (4) meet that the period of time of the corresponding reference technology coincides with the period of time of the fixed output quantity, and all the output distance functions satisfy that the period of the corresponding reference technology is equal to the period of the fixed input quantity. Third, the HMTFP index is well-defined even under the assumption of variable returns to scale. This implies that, for example, in the context of one input and one output, the HMTFP index always matches to the well-known form $\left(y^{t+1}/x^{t+1}\right)/\left(y^t/x^t\right)$ regardless the returns to scale assumed for the production possibility sets in periods t and $t+1$.

An additional advantage of the definition of the HMTFP index is that allows determining measures of output change, $QI^{t,t+1}\left(x^t,y^t,x^{t+1},y^{t+1}\right)$, and input change, $XI^{t,t+1}\left(x^t,y^t,x^{t+1},y^{t+1}\right)$, which can be useful from a managerial point of view, especially in education, as we will show in the next section.

As for the interpretation of the numerical values of the numerator, denominator and the ratio associated with the HMTFP index, $QI^{t,t+1}\left(x^t,y^t,x^{t+1},y^{t+1}\right)>1$ indicates an increase in outputs from period t to period $t+1$ and $QI^{t,t+1}\left(x^t,y^t,x^{t+1},y^{t+1}\right)<1$ a decline. The values related to $XI^{t,t+1}\left(x^t,y^t,x^{t+1},y^{t+1}\right)$ can be interpreted in the same manner with respect to the inputs. Regarding the HMTFP index, values greater than one for $HM^{t,t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right)$ indicate TFP growth, while values lower than one point to decreases in TFP.

Regarding the possibility of decomposing the HMTFP index, there exist several alternative possibilities. Without the intention of being exhaustive, Lovell (2003) seems to be the first contribution that presents an attempt to decompose the index into its sources. More recently, O'Donnell (2012a) has introduced a general decomposition, which is valid for any 'multiplicatively complete' index[6], that can be particularly applied to the HMTFP index:

$$HM^{t,t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \left(\frac{TFP^{t+1*}}{TFP^{t*}}\right) \cdot \left(\frac{TFPE^{t+1}}{TFPE^{t}}\right), \quad (5)$$

where $TFP^{k*} = \max\left\{\left[\frac{D_O^t\left(x^t, y\right) \cdot D_O^{t+1}\left(x^{t+1}, y\right)}{D_I^t\left(x, y^t\right) \cdot D_I^{t+1}\left(x, y^{t+1}\right)}\right]^{1/2} : (x, y) \in T^k\right\}$, $k = t, t+1$, represents TFP at the point of maximum productivity in period k, and $TFPE^k = \frac{\left[\frac{D_O^t\left(x^t, y^k\right) \cdot D_O^{t+1}\left(x^{t+1}, y^k\right)}{D_I^t\left(x^k, y^t\right) \cdot D_I^{t+1}\left(x^k, y^{t+1}\right)}\right]^{1/2}}{TFP^{k*}}$, is the so-called TFP efficiency and represents an overall measure of DMU performance as the ratio of observed TFP in period k to the maximum TFP possible using the available technology in period k.

The first component in parenthesis on the right-hand side of expression (5) can be interpreted as a measure of the change in the maximum TFP over time, which is a natural measure of technical change. The second component may be interpreted as a measure of overall efficiency change (see the discussion in O'Donnell, 2012a).

In this paper, we apply the O'Donnell decomposition because it is justified from a theoretical standpoint. The computation of the HMTFP index in our empirical analysis is implemented using the software DPIN 3.0 [7].

[6] They are indices that can be written as the ratio of an index of aggregated output change over an index of aggregated input change where the aggregator functions are non-negative non-decreasing linearly-homogeneous scalar functions.

[7] This software is available at http://www.uq.edu.au/economics/cepa/dpin.htm.

Finally, following Berg et al. (1992), who defined a base-period Malmquist index fixing a baseline period technology; it is possible to define the HMTFP index in a similar way. In particular, in our study, we apply a HMTFP index where the academic year 2009-2010 is considered as the reference year for all the calculations, since that year was not affected by the subsequent cuts in the public education budget.

3. Data and variables

3.1. Public education and the economic crisis in Catalonia

The educational system in Spain is decentralized in 17 regions, the so called *Comunidades Autónomas*, which have full competences for funding and monitoring public and private grant-aided schools. Central Government holds law powers to establish a minimum common educational framework for the whole country. Public schools are funded by the taxpayers and managed by the Department of Education of each region. Private schools include both the purely private schools and the private grant-aided schools.

The economic reality in Spain is currently the subject of much debate as the government rationalized education spending from 2010 up to 2015, coinciding with the global economic crisis, through more than 11% budget cut to save 3,000 million euros in ordet to reduce the public deficit of Public Administrations. This cutback was implemented using certain measures, such as increasing the student-teacher ratio, expanding the range of increases in university fee or suspending teacher replacements.[8] The percentage of GDP devoted to Education in Spain felt more than the country's wealth, from 5.03% of GDP in 2008 to 4.79% in 2011, reaching a minimum of 4.28% in 2014 (Martínez, 2011).

The main consequence of the impact of the crisis is the worrying rise in the pupil-teacher ratio and the lack of resources in the public education sector. The differences in per capita expenditure in education are much greater than can explain any reasonable index of cost per inhabitant. Under these conditions, this basic public service is not likely to be provided at the same level in the regions, despite being very relevant for equal opportunities. The consequences were especially drastic in Catalonia, where the

[8] Spanish Law for the Improvement of Educational Quality (*Ley Orgánica 8/2013, de 9 de diciembre, para la mejora de la calidad educativa (LOMCE)*).

investment in public education in terms of GDP remained below the Spanish average. Comparing 2007 and 2013, Pérez et al. (2015) conclude that Extremadura and Andalusia allocate the highest percentage of GDP to Education (6.5% and 5.7%, respectively in 2013) with the lowest percentages in Spain we can be found in Catalonia and Madrid (2.6% and 3.4%, respectively in 2013) that are the two regions with the highest GDP in Spain.

Public primary education in Catalonia is compulsory since six years up to sixteen years old and free since three years old. It is divided into three two-year cycles totaling six academic courses, which are usually taken between the ages of six and twelve. At the end of this stage (sixth year of primary education) students are tested on their basic skills in mathematics, Catalan, Spanish and a foreign language (English). The Evaluation Council of the Education System in Catalonia (*Consell Superior d'Avaluació del Sistema Educatiu*) and the relevant bodies of the Regional Educational Areas (REAs) collaborate in conducting a comprehensive standardized diagnostic assessment on students[9].

3.2. Sample description

To analyze the impact of the economic crisis on schools' productivity, we use data from the abovementioned Evaluation Council. The data set is composed of fully public primary schools that are classified as facing disadvantaged conditions.[10] The relevant unit of observation is the school and it includes information from the academic year 2009 up to 2013, covering a wide period to analyze the antecedents and consequences of the crisis. Using this information, we build a balanced panel data set composed of 298 public primary schools over five years in Catalonia. Table 1 summarizes the number of students and schools per year. As can be seen, our balanced panel is composed of 1,490 schools in total, 298 schools per year, encompassing 430,059 pupils in eight REAs.

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[9] The public education system in Catalonia is divided into education areas (REAs). These REAs act as local agencies in each area and each one has a team of Inspectors of Education who provide specific assistance to schools and help them conduct the basic skill tests. We will divide our estimations by REA in order to identify statistical differences in the effect of the crisis on schools' performance depending on their location.

[10] According to the *Dirección General de Profesorado y Personal de Centros Públicos* from Catalonia, these schools are immersed in environments with socio-economic and socio-cultural characteristics especially disadvantaged. This will depend, among other factors, on the number of pupils with special educational needs, the proportion of immigrants or the number of pupils with late enrollment in the system.

Table 1. Sample decomposition

Academic year	2009	2010	2011	2012	2013	Total
Students	83,599	84,580	86,833	88,149	86,898	430,059
Schools	298	298	298	298	298	1,490

Source: Own elaboration

Prior to setting up the final sample showed in Table 1, a test for outliers was performed. It is well-known that extreme points can influence the shape of the estimated production frontier and introduce bias in the TFP changes. In relation to frontier analysis, two influential contributions are those of Andersen and Petersen's (1993) super-efficiency coefficient and Wilson's paper (1993). Therefore, through the super-efficiency test the influential units found in the sample are removed and the efficiency measures re-estimated. Moreover, following Prior and Surroca (2010), this procedure is continued as long as the null hypotheses of equality between successive efficiency scores cannot be rejected. By doing so, about 3% of the schools in the total sample turn out to be outliers and are therefore eliminated.

3.3. Variables

The dataset also provides a large amount of information on students' educational achievement in the basic skill test, on their families, on the educational environment, and on school management. Table 2 contains the definitions of the variables included in the current analysis.

Following the literature on efficiency in education we use as outputs (*Y*) the results from standardized aptitude tests in four subjects; Catalan, Spanish, English and Mathematics. As Hoxby (2000) states, the most important reason to choose this proxy as an output could be that both policy makers and parents use this criterion to evaluate school performance. In terms of inputs, the vast literature analyzing the determinants of educational outcomes distinguishes two main blocks of factors. The first focuses primarily on the role of the students' family background, including socioeconomic status (SES) (Sirin, 2005). The second block covers aspects related to school factors and teaching practices (Reynolds et al., 2014). In this paper, we include three inputs that are directly involved with student learning (*Teach/Stud*, *Expenses/Stud* and *SES).*

Table 2. Definition of variables

Label		Variable	Description
Outputs	Y_1	**Catalan**	Average grade in Catalan in the general sixth grade test.
	Y_2	**Spanish**	Average grade in Spanish in the general sixth grade test.
	Y_3	**English**	Average grade in English in the general sixth grade test.
	Y_4	**Mathematics**	Average grade in mathematics in the general sixth grade test.
Inputs	X_1	**Teach/stud**	Number of teachers per 100 students.
	X_2	**Expenses/Stud**	Expenses per student.
	X_3	**SES**	Average Mothers' educational level. 0. No education. 1. Primary education. 2. Low Secondary Education. 3. Professional training (middle level) 4. A-Levels. 5. Professional training (high level). 6. University degree. 7. Postgraduate. 8. Doctorate.

Source: Own elaboration.

Teach/stud represents the number of academic staff working in the school per 100 students. The literature reveals that the number of personnel employed is a good proxy for human capital in education institutions (e.g., Haelermans, De Witte and Blank, 2012; Thieme *et al.,* 2012; Brennan *et al.,* 2014). *Expenses/Stud* refers to the operational expenses per student in each school excluding teachers' costs. A higher level of spending should lead to better outcomes (Agasisti, 2014; Brennan *et al.*, 2014). Lastly, *SES* proxies the socio-economic level of the families. It is expected that higher values of this index should lead to better educational outcomes (e.g., Thieme *et al.,* 2013; Agasisti, 2014; Crespo-Cebada *et al.,* 2014). We use the mothers' educational level as the literature has revealed that the effect on academic achievement is determinant (Kong and Fu, 2012).

Table 3 summarizes the main descriptive statistics of input and output variables. As can be seen, students' test scores are, on average, higher than 50 points out of 100 in the basic skills tests. At these levels of education most students usually pass, although there are some who do not reach scores of 50 in English. In addition, teacher per student ratio is decreasing over the analyzed period, starting at 9.906 in 2009 until 8.508 in 2013, on average. We find a minimum of 7.330 in 2012, suggesting that budgetary constraints that occurred during the crisis also affected the number of teachers employed. The variable expenditure follows a similar path, decreasing over the period, showing a slight rise in the last academic year, 2013. Finally, SES is stable over time on average.

Table 3. Descriptive statistics input/output variables

Variable		2009	2010	2011	2012	2013
Catalan	Mean	62.929	66.797	75.679	61.289	67.221
	St. Dev.	9.236	9.925	11.247	9.325	8.817
Spanish	Mean	60.603	63.043	67.353	64.459	68.046
	St. Dev.	9.970	10.719	9.225	8.410	8.231
English	Mean	56.760	69.796	63.010	60.031	62.846
	St. Dev.	11.323	10.460	10.027	12.146	11.919
Mathematics	Mean	68.340	71.869	70.454	70.610	74.208
	St. Dev.	9.628	10.001	10.977	10.980	8.720
Teach/Stud	Mean	9.906	9.753	9.383	7.330	8.508
	St. Dev.	3.043	2.887	2.920	2.507	4.352
Expenses/Stud	Mean	43.621	41.907	32.211	32.319	35.322
	St. Dev.	28.477	27.094	23.552	21.023	20.977
SES	Mean	4.345	4.362	4.397	4.412	4.345
	St. Dev.	0.822	0.825	0.832	0.835	0.822
	N	298	298	298	298	298

Source: Own elaboration.

4. Analysis of Results

This section progressively presents the empirical results of the HMTFP index. All computed results are feasible and TFP interpretations are offered together with frontier components. Specifically one can see the TFP behavior of the analyzed DMU in the HMTFP index result and also obtain efficiency frontier information both for output and input orientations from the numerator and denominator of the index. When used for managerial decision making, these features of the HMTFP index significantly improve upon the properties of a standard Malmquist index.

The differences between the successive HMTFP indexes are assessed through a Li test (see Li, 1996; and Kumar and Russell, 2002). This non-parametric statistical test compares two unknown distributions using kernel densities. Its advantages are twofold. First, the Li test statistic is valid for dependent and as well as for independent variables (Kumar and Russell, 2002). Second, in contrast to most statistical tests, the Li test is not based on mean or median comparisons, but instead compares two entire distributions to each other. Thus, by means of the Li test p-value, the null hypothesis of equality of distributions can be rejected or not.

4.1. The Hicks-Moorsteen TFP index

The HMTFP index is computed by establishing a base-year (i.e., 2009, one year before the drop in the public education expenditure) and assuming variable returns to scale. Thus, while the reference year is not allowed to change, the sample can experience efficiency and technological changes. In general, this index must be interpreted with respect to the base-year. Tables 4 and 5 present the descriptive statistics associated with the HMTFP index and its decomposition. The results of the HMTFP index are straightforward and easy to understand, values higher/lower than unity indicate a better/worse TFP than the reference year. The index's distribution is also shown through percentile results, which have the advantage of avoiding the biases that top/bottom DMUs can create in the mean values.

Table 4. Output-Input changes and HMTFP index (DMUs = 298)

Component	Years	Percentiles			Mean	St. Dev
		25	Median	75		
Output change	2009-10	1.022	1.082	1.158	1.094	0.143
	2009-11	1.060	1.136	1.224	1.151	0.165
	2009-12	0.971	1.030	1.105	1.045	0.127
	2009-13	1.001	1.070	1.175	1.097	0.159
Input change	2009-10	0.980	0.998	1.014	0.994	0.057
	2009-11	0.904	0.955	0.989	0.939	0.082
	2009-12	0.826	0.880	0.934	0.880	0.092
	2009-13	0.740	0.789	0.850	0.797	0.109
HMTFPC	2009-10	1.024	1.093	1.179	1.103	0.149
	2009-11	1.128	1.206	1.304	1.235	0.203
	2009-12	1.084	1.186	1.309	1.201	0.206
	2009-13	1.233	1.368	1.533	1.402	0.288

Source: Own elaboration.

Analyzing the HMTFP index, the results from Table 4 reveal a positive TFP index during the whole period (at all percentile levels). The TFP increases with respect to the base year due to the expected shortcut in inputs, as the input change remains below the unity during the period. However, surprisingly, we also observe a positive change in outputs (i.e., academic achievement in the basic skill tests) over the period. This finding is very interesting as it is showing that despite the lack of resources and the drop in the public expenditure devoted to education during the crisis period (see Figure 1) most vulnerable schools improved their academic achievement in spite of losing resources. This result is complemented with the HMTFP decomposition showed in Table 5.

Table 5. HMTFP index decomposition (DMUs = 298)

Component	Years	Percentiles			Mean	St. Dev
		25	Median	75		
HMTFPC	2009-10	1.024	1.093	1.179	1.103	0.149
	2009-11	1.128	1.206	1.304	1.235	0.203
	2009-12	1.084	1.186	1.309	1.201	0.206
	2009-13	1.233	1.368	1.533	1.402	0.288
Technological change	2009-10	1.071	1.100	1.133	1.103	0.053
	2009-11	1.265	1.391	1.461	1.378	0.153
	2009-12	1.110	1.202	1.297	1.192	0.114
	2009-13	1.327	1.544	1.686	1.560	0.276
Efficiency change	2009-10	0.918	0.995	1.074	1.002	0.141
	2009-11	0.794	0.871	0.982	0.904	0.163
	2009-12	0.921	0.986	1.066	1.009	0.149
	2009-13	0.770	0.888	1.055	0.924	0.244

Source: Own elaboration.

One can note that, at the median level (as well as any of the percentiles), public schools from specially disadvantage environments in Catalonia experience a substantial and progressive TFP growth over the period (36.8% at median level), compared to 2009, because of a consistent positive technological change.

A potential factor for explaining the positive technological change observed during this period could be an educational program that was released in 2010. Despite the budgetary constraints during the crisis, the Department of Education in Catalonia deployed an educational plan to support and advise public schools, through specific assistance and teachers training from the Inspectors of Education. In addition, this plan created a network to foster the exchange of knowledge and experiences between schools. The actions of this plan did not directly involve higher resources to schools, but provided them with support and assistance that would be able to foster academic performance (positive output change revealed in Table 4). Nevertheless, Table 5 also reveals a negative change in efficiency, in the sense that not all schools adopted the educational programme. The Li tests confirm these findings and prove that the HMTFP index and its component are significantly different over time, showing a significant rise in the TFP indicator.[11]

Summarizing, the results reveal that in times of crisis schools that are more vulnerable in terms of resources improved their academic achievement in spite of losing resources, so the HMTFP index was positive. The decomposition of the index points out that there was a positive technological change (probably consequence of the abovementioned), but nevertheless it indicates a negative change in efficiency, in the sense that not all schools adapted to this change. The advantage of this HMTFP index in terms of informative content is that policy makers may better establish their goals on resource allocation through a monitoring system by comparing the evolution in terms of efficiency and technological changes over time and, at the same time, it is possible to reach conclusions on output and input growth separately.

4.2. Area by area analysis

In order to complement our findings from the whole sample we split the results by educational area (REA). It is worth recalling that these REAs act as local agencies that depend on the Department of Education in Catalonia. They are managed by a group of Inspectors of Education who provide specific assistance to schools and help them conduct the basic skill tests. Therefore, we will divide our estimations by REA in order to identify statistical differences in the effect of the crisis on schools' performance depending on their geographical location. While general results from previous section are probably most relevant for researchers, policy makers and schools principals are arguably more interested in comparing a particular school or a particular area to their neighbors. For this second type of analysis and under VRS, the feasibility of the HMTFP indicator offers a clear advantage over the standard Malmquist index. To illustrate this approach, we maintain 2009 as the baseline, and analyze the evolution of the TFP index by educational area. Table 6 collects the main results.[12]

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[11] We did not display the detail of the Li tests performed, but they are available from authors upon request.
[12] To save space, we only show the median and standard deviation. Full information is available from authors upon request.

Table 6. HMTFP index decomposition over time (by area)

Area	Years	Output change		Input change		HMTFP		Techn. change		Eff. change	
		Med.	**St.Dev**	**Med.**	**St.Dev**	**Med.**	**St.Dev**	**Med.**	**St.Dev**	**Med.**	**St.Dev**
1.Baix Llobregat (n=23)	2009-10	1.098	1.172	0.986	1.001	1.142	1.182	1.071	1.096	1.045	1.113
	2009-11	1.166	1.234	0.917	0.968	1.280	1.394	1.422	1.586	0.907	0.964
	2009-12	1.032	1.135	0.847	0.890	1.243	1.338	1.293	1.316	0.992	1.042
	2009-13	1.095	1.176	0.785	0.841	1.397	1.478	1.391	1.641	1.014	1.105
2.Counties of Barcelona (n=56)	2009-10	1.079	1.186	0.998	1.028	1.097	1.207	1.083	1.112	1.004	1.104
	2009-11	1.141	1.230	0.936	0.973	1.237	1.395	1.416	1.461	0.887	1.016
	2009-12	1.035	1.112	0.874	0.915	1.188	1.324	1.210	1.309	0.992	1.061
	2009-13	1.084	1.192	0.798	0.834	1.395	1.552	1.460	1.635	0.922	1.077
3.Vallés Occidental (n=42)	2009-10	1.090	1.139	1.000	1.016	1.091	1.165	1.103	1.126	0.973	1.047
	2009-11	1.156	1.226	0.964	0.992	1.228	1.270	1.422	1.513	0.861	0.969
	2009-12	1.009	1.076	0.891	0.949	1.156	1.243	1.188	1.298	0.947	1.029
	2009-13	1.085	1.157	0.808	0.857	1.339	1.443	1.578	1.691	0.859	0.997
4.Barcelona city (n=47)	2009-10	1.085	1.160	0.992	1.000	1.101	1.184	1.102	1.156	0.989	1.077
	2009-11	1.136	1.208	0.978	1.002	1.184	1.295	1.423	1.476	0.831	0.901
	2009-12	1.023	1.095	0.839	0.911	1.247	1.390	1.212	1.314	1.039	1.106
	2009-13	1.063	1.156	0.773	0.804	1.353	1.502	1.517	1.700	0.917	1.063
5.Girona (n=30)	2009-10	1.098	1.178	1.001	1.017	1.096	1.191	1.118	1.147	0.996	1.058
	2009-11	1.116	1.220	0.974	1.016	1.169	1.309	1.369	1.453	0.943	1.031
	2009-12	1.045	1.121	0.932	0.978	1.138	1.219	1.138	1.267	0.974	1.022
	2009-13	1.062	1.203	0.784	0.856	1.319	1.576	1.648	1.734	0.813	0.976
6.Lleida (n=18)	2009-10	1.044	1.094	0.995	1.008	1.061	1.143	1.129	1.181	0.920	0.983
	2009-11	1.084	1.255	0.964	0.986	1.150	1.307	1.360	1.379	0.947	1.044
	2009-12	1.024	1.102	0.858	0.955	1.200	1.300	1.134	1.240	1.011	1.181
	2009-13	1.100	1.185	0.774	0.824	1.457	1.584	1.614	1.706	0.876	1.020
7.Maresme-Vallés Oriental (n=35)	2009-10	1.072	1.120	0.998	1.012	1.081	1.139	1.097	1.115	0.991	1.053
	2009-11	1.104	1.151	0.936	0.951	1.204	1.277	1.422	1.527	0.839	0.925
	2009-12	1.022	1.041	0.868	0.908	1.149	1.204	1.187	1.247	0.990	1.049
	2009-13	1.027	1.105	0.814	0.856	1.277	1.443	1.497	1.586	0.874	1.055
8.Tarragona (n=47)	2009-10	1.083	1.157	1.003	1.029	1.087	1.157	1.094	1.132	0.996	1.063
	2009-11	1.128	1.225	0.964	0.991	1.205	1.309	1.251	1.396	0.923	1.102
	2009-12	1.056	1.105	0.914	0.970	1.166	1.324	1.212	1.300	0.984	1.092
	2009-13	1.109	1.213	0.771	0.885	1.410	1.614	1.559	1.722	0.890	1.051

Source: Own elaboration.

As can be seen, these findings by area can be interpreted on average similarly to the previous one. The Li tests confirm that the TFP growth shown during the period is statistically different, being significantly higher in the last year (2013) compared to the ones obtained during all other periods. The differences in outputs and the technological change are also evident and significant during all the years in all REAs. The only difference is that the negative efficiency change identified for the whole sample, at median level, is not evident for all REAs during the period. Specially, the positive efficiency change identified in schools from the cities of Barcelona and Lleida, in the third year, and from *Baix Llobregat* in the last year, could be associated with better managerial practices than the rest of schools in other REAs. Particularly, schools from *Baix Llobregat* show a systematic recovery trend from 2011 onwards, meanwhile the pattern in the rest of REAs is not as consistent as the observed in REA 1.

We think that micro information, at school and area level, contained in Table 6 is doubtlessly useful to allocate new resources in subsequent periods and for detecting best practices. For example, the HMTFP index shows that for the whole period productivity growth was 18 points higher in REA 6 – Lleida than in REA 7 – Maresme Vallés Oriental. Additionally, the HMTFP index grew homogeneously during the period analyzed in the City of Barcelona, without showing great ups and downs from one academic year to another, unlike the rest of educational areas, where we find less uniformity in the growth rate of the change in productivity indicator.

In conclusion, we can observe that the financial crisis has caused a reduction of the public expenditure devoted to education in a generalized way in all educational areas. Despite this, the most vulnerable schools have made an effort to maintain and even improve the academic performance of their students. This effort to improve educational performance is evident in all areas and has led to an increase in productivity during the analyzed period. However, the change in technical efficiency has been negative, except for the schools located in Barcelona, *Lleida* and *Baix Llobregat* that stand out for their good practices of educational management, thus showing a positive change in efficiency at the end of the period, sign of recovery.

5. Conclusions

The present study applies the HMTFP change index as an alternative model to assess productivity changes that overcomes the pitfalls of other indexes such as Malmquist index. The HMTFP index is defined as a ratio of an aggregate output-quantity over an aggregate input-quantity index. More precisely, it measures the change in output quantities in the output direction and the change in input quantities in the input direction, instead of exclusively adopting an input- or output-orientation as Malmquist indexes usually do. The TFP characteristics of the HMTFP index solve the limitations of the traditional Malmquist productivity index in the presence of VRS. Furthermore, this HMTFP index is well-defined under general assumptions of variable returns to scale and strong disposability. However, in spite of its attractive properties, the HMTFP has been scarcely empirically applied, and we do not find any application in the education context.

Consequently, we apply the HMTFP index in a sample of 298 public schools from Catalonia using a rich balanced data set covering five academic years, from 2009-2010 to 2013-2014,[13] representing a suitable period to analyze the antecedents and consequences of the economic crisis on productivity changes. We put special emphasis in those schools located in poorest neighborhoods, as they are the most affected by budget cuts.

The HMTFP can be directly interpreted as a TFP index analyzing its numerator (outputs) and denominator (inputs). In our empirical analysis results are very interesting because they reveal that in crisis times schools were more vulnerable as consequence of the shortcut in resources. However, academic achievement grew during the period leading to a HMTFP index significantly greater than one. The decomposition of the index indicates that there has been a positive technological change (probably consequence of a special educational program released in 2010 in Catalonia) but nevertheless it reveals a negative change in efficiency, in the sense that not all schools adapted to this change.

We also find appealing results when distinguishing by educational areas in the region of Catalonia. The above findings remain, however it is interesting to see, especially in the

[13] For simplicity, we will refer to each academic year by mentioning the first natural year (for instance, 2009 to refer to 2009/2010).

differences after four academic years (2009/2013), the different paths followed in each educational area, perhaps associated with good management practices. It is also relevant to highlight that only one area, *Baix Llobregat*, does not experience a negative change in efficiency at the end of the analyzed period, rather the opposite.

The advantages of the proposed tool for the educational context are various. First, this Hicks-Moorsteen type index, which is scarcely used, solves known problems of TFP measurement in the presence of VRS, which represents a common scenario when comparing the performance of education institutions. Indeed, under weak assumptions of strong disposability and VRS, this index is always feasible. This property is crucial for benchmarking analysis as unit-specific results have to be provided. Second, policy makers may better establish their goals on resource allocation through a monitoring system by comparing the evolution in terms of efficiency and technological changes over time and, at the same time, it is possible to reach conclusions on output and input growth separately.

Acknowledgments

J. Aparicio thanks the financial support from the Spanish Ministry for Economy and Competitiveness (*Ministerio de Economia, Industria y Competitividad*), the State Research Agency (*Agencia Estatal de Investigacion*) and the European Regional Development Fund (*Fondo Europeo de Desarrollo Regional*) under grant MTM2016-79765-P (AEI/FEDER, UE). L. López-Torres acknowledges the financial support of the FPU grant (number 12/01341). Daniel Santín thanks the financial support of the Spanish Ministerio de Ciencia e Innovación (ECO2014-53072-P).

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